Math 2142: Calculus II

Credit hours: 4 credit hours
Prerequisites: MATH 2141 with a grade of C or better

Course Description

This course covers the calculus of logarithmic, exponential, trigonometric, inverse trigonometric and hyperbolic functions. Some methods of integration are covered, including integration by parts and numerical methods. L’Hospital’s rule, improper integrals, infinite series and the calculus in polar coordinates are also discussed.

Course Objectives

1. Expand on the foundation of knowledge and skills established in Calculus I
2. Further establish the fundamental theorems and applications of the calculus of single variable functions
3. Expand on the concepts, properties and aspects of the differential and integral calculus of single variable functions
4. Provide students with further mathematical tools necessary for more advanced STEM fields

Learning Outcomes

1. Calculate limits, derivatives, and indefinite integrals of various functions of a single variable including transcendental functions
2. Use the properties of limits and the derivative to analyze graphs of various functions of a single variable including transcendental functions
3. Employ the principles of the differential calculus to solve optimization problems, related rates exercises, and other applications
4. Utilize advanced techniques of integration to calculate the area of regions in the plane and the volume and surface area of solids of revolution
5. Apply the techniques of numerical integration to solve both pure and applied mathematics problems
6. Calculate the limit of infinite sequences
7. Determine the absolute or conditional convergence of series
8. If possible, calculate or estimate the sums convergent series
9. Investigate the infinite series representation of functions of a single variable
10. Apply the concepts of differential calculus to parametric equations
11. Apply the concepts of differential calculus to functions in polar coordinates

Course Topics

I. Logarithmic and Exponential Functions
   A. The integral definition for \( y = \ln(x) \)
   B. The exponential function \( y = e^x \) as the inverse of \( y = \ln(x) \)
   C. Derivatives and Integrals of logarithmic and exponential functions
   D. Logarithmic differentiation
   E. Functions involving bases other than e
   F. Some limits equal to e
II. Trigonometric Functions and their Inverses
   A. Derivatives and integrals of trigonometric functions
   B. Inverse trigonometric functions
      1. Definitions
      2. Graphs
      3. Derivatives
      4. Integrals

III. Hyperbolic Functions
   A. Definitions
   B. Graphs
   C. Derivatives
   D. Integrals
   E. Inverse hyperbolic functions*

IV. Methods of Integration
   A. Integration by parts
   B. Integration by partial fraction decomposition
   C. Integration by trigonometric substitution*
   D. Integration tables
   E. Numerical integration
      1. The Trapezoidal Rule
      2. Simpson’s Rule

V. Limits
   A. Indeterminate forms
   B. L’Hospital’s Rule
      1. Direct application of L’Hospital’s Rule
      2. Rewriting indeterminate forms prior to applying L’Hospital’s Rule

VI. Improper Integrals
   A. Improper integrals involving infinity
   B. Improper integrals involving closed intervals
   C. Improper integrals involving mixed type

VII. Sequences and Series
   A. Sequences
      1. Definition
      2. Notation
      3. Convergence
      4. Divergence
   B. The definition of a partial sum
   C. Series
      1. Definition
      2. Notation
      3. Convergence as a limit of a sequence of partial sums
      4. Divergence
   D. Types of series
      1. Geometric series
      2. Telescoping series
3. P-series and the Harmonic Series
4. Alternating Series
E. Tests for Convergence or Divergence
   1. $n^{th}$-term test
   2. Geometric series formula
   3. Telescoping series formula
   4. $p$-series test
   5. Alternating series test
   6. Integral test
   7. Ratio test
   8. Root test
   9. Comparison test
   10. Limit comparison test
F. Taylor and Maclaurin polynomials
G. Power Series
   1. Definition
   2. Representation of functions by power series
   3. Taylor and Maclaurin series

VIII. Polar Coordinates
   A. Definition of polar coordinates
   B. Conversion to and from rectangular coordinates
   C. Polar graphs
   D. Finding points of intersection for polar graphs
   E. Calculating areas in polar coordinates
   F. Conic Sections*

IX. Parametric Equations in the Plane
   A. Definition of a parametric equation
   B. Graphs of parametric equations
   C. Elimination of the parameter
   D. Finding slopes and tangent lines
   E. Arc length

*Optional