P. 47 Note that in Example 5 the symbol (~) is translated as "it is not true that..."

P. 50 To help understand "negations of statements" replace the word "some" with "at least one" in statements **A** and **B** below:

| | |
|---|---|
| Statement **A:** **Some birds can fly.**<br>Equivalent Statement: At least one bird can fly.<br><br>Negation: Not (even) one bird can fly.<br>Equivalent Negation: **No birds can fly.** | Statement **B:** **Some students are not motivated.**<br>Equivalent Statement: At least one student is not motivated.<br><br>Negation: **Not** (even) one student is **not** motivated.<br>① ②<br>Equivalent Negation: **All students are motivated**. |

Having the two "nots" in the negation of statement **B** made the statement quite awkward to read. The standard way to simplify this is to replace both "nots" with "All... are..."

If we had changed the "not (even) one student..." to "no students..." as we did in the negation of statement **A**, we would still be left with an awkward statement containing the two negatives, "no" and " not."

P. 51 Know these equivalent statements.

P. 52 Conjunction (and) table --memory aid: the only true case is the "T ∧ T" case where both statements are true (T).

P. 53 Disjunction (or) table -- memory aid: the only false case is the "F ∨ F" case where both statements are false (F).

P. 56 For an alternative and often simpler method in constructing a truth table, consider the following compound statement: ~p ∨ q.

**Step 1**
Instead of listing the four possible truth-value cases for statements "p" and "q" on the left of the statement (as your text shows), simply list them under each "p" and "q" as they occur in the statement.

**Step 2**
Next, determine the "~p" column under the "~" heading.

**Step 3**
Determine the answer column using the "~" column and the "q" column, along with the definition of "∨ " (or).

| ~ | p | ∨ | q |
|---|---|---|---|
| | T | | T |
| | T | | F |
| | F | | T |
| | F | | F |

Step ①

| ~ | p | ∨ | q |
|---|---|---|---|
| F | T | | T |
| F | T | | F |
| T | F | | T |
| T | F | | F |

Step ②

Answer

| ~ | p | ∨ | q |
|---|---|---|---|
| F | T | **T** | T |
| F | T | **F** | F |
| T | F | **T** | T |
| T | F | **T** | F |

Step ③

P. 58 Suppose we are asked to construct a truth table for the negation of the above problem: ~(~p ∨ q). Using this method, you would follow the same three steps as illustrated above and then simply take the negation of the answer column in Step ③ to obtain "F T F F."

P. 61 First **DeMorgan Law**: **~(p ∧ q) ≡ ~p ∨ ~q** -- memorize.

P. 65 Second **DeMorgan Law**: **~(p ∨ q) ≡ ~p ∧ ~q** -- memorize.

P. 69

| | | |
|---|---|---|
| The implication:<br>If p, then q.<br>(or)<br>p implies q.<br>(or)<br>p → q | can also be written as: | p only if q.<br>(or)<br>p is a sufficient condition for q. |

| | | |
|---|---|---|
| The converse:<br>If q, then p.<br>(or)<br>q implies p.<br>(or)<br>q → p | can also be written as: | p, if q.<br>(or)<br>p is a necessary condition for q. |

P. 73 Implication Table --memory aid: the only false case is the "T → F" case where the hypothesis is true (T) and the conclusion is false (F). The order is significant.

P. 80 The implication (p→q) and the contrapositive (~q →~p) are logically equivalent. Therefore, they are interchangeable statements; one may be substituted for the other.

P. 83 The biconditional $(p \leftrightarrow q)$ means “p if and only if q” and also “p is necessary and sufficient for q.”

P. 84 Biconditional Table -- memory aid: true cases are the “$T \leftrightarrow T$” case and “$F \leftrightarrow F$” case where both statements are either true (T) or both are false (F). Knowing that a biconditional statement is the conjunction of a statement and its converse, you are able to derive the biconditional table (without memorizing it) by constructing a truth table for $(p \to q) \wedge (q \to p)$. Using the alternative method described earlier we have:

NOTE: In step ① the truth values for “p” and “q” were each listed twice since there are two “p’s” and two “q’s” in the statement.

NOTE: Step ② and step ③ could have been interchanged. Either one could be done before the other.

Step ④↓

Step ② Step ③

| (P | → | q) | ∧ | (q | → | p) |
|---|---|---|---|---|---|---|
| T | T | T | **T** | T | T | T |
| T | F | F | **F** | F | T | T |
| F | T | T | **F** | T | F | F |
| F | T | F | **T** | F | T | F |

Step ①

P. 86 The implication $(p \to q)$ is also logically equivalent to the disjunction “not p or q” $(\sim p \vee q)$, as well as the contrapositive $(\sim q \to \sim p)$.

NOTE: Become familiar converting a disjunction to an implication, and vice versa: (1) negate the statement on the left; (2) interchange the “$\vee$ ” with the “$\to$”; and (3) copy the statement on the right as is. For example, to change “$p \vee \sim q$” to an implication, (1) negate “p” to “~p”; (2) replace “$\vee$ ” with “$\to$”; and (3) copy “~q”. Doing so, we obtain “$\sim p \to \sim q$.”

P. 106 (#2d) Using the alternative method (page 56) we may construct the truth table for this problem (argument) as follows: $[(p \to q) \wedge \sim p] \to q$.

Step ⑤

Step ④

| [(p | → | q) | ∧ | ~ | p] | → | q |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | **T** | T |
| T | F | F | F | F | T | **T** | F |
| F | T | T | T | T | F | **T** | T |
| F | T | F | T | T | F | **(F)** | F |

Step ② Step③

Step ①

NOTE: In step ① the truth values for “p” and “q” were each listed twice since there are two “p’s” and two “q’s” in the statement.

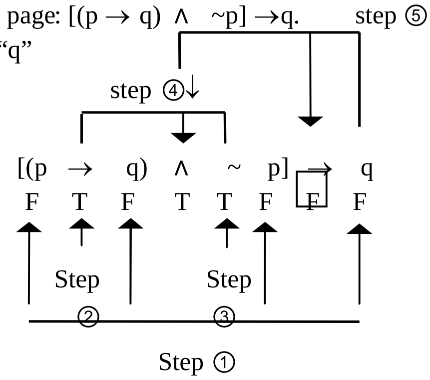
NOTE: Step ② and Step ③ could have been interchanged. Either one could have been done before the other.

NOTE: Since Case 4 is false (F), the above compound statement (argument) is not a tautology; and therefore, not valid.

More on Truth Tables -- Analyzing Just One Case

Suppose we are asked to give the truth value corresponding to just one of the four cases. You could of course construct the entire truth table and then select the one truth value which corresponds to the particular case being asked for; however, you could also try to determine the value by analyzing just that one case.

To illustrate, let's return to the compound statement given on the previous page: [(p → q) ∧ ~p] →q. Suppose we are asked to determine its truth value for Case 4 only, where "p" is "F" and "q" is "F." Using the same five steps, we are able to conclude that the statement is "F" when "p" is "F" and "q" is "F," without constructing the entire truth table.




NOTE: Step ② and step ③ could have been interchanged.

Now let's **reverse** the process. Suppose it is given that the truth value for the above compound statement is "F" and we are asked to determine the individual truth values of "p" and "q."

We then have: [(p →q) ∧ ~p] →q (**F**)

hypothesis: [(p →q) ∧ ~p]  conclusion: q

Since this compound statement is an implication, the only way it can result in an "F" value is when the hypothesis is "T" and the conclusion is "F".

We now have: [(p →q) ∧ ~p] →q (**F**)

[(p →q) ∧ ~p] is **T**; q is **F**

Thus, the truth value of "q" is "F." We now need to find "p." Since the hypothesis is a conjunction (∧ ) and its value is "T," the only way this could be is when statement "p →q" and statement "~p" are both "T." Hence, since "~p" is "T," the truth value of "p" must be "F." Therefore, we have found that the truth value of "p" is "F" and the truth value of "q" is also "F."

If you look back at the complete truth table, you will see that case 4 is the only case where the given compound statement yields a value (F) different from the others. This is why this example was selected. With any of the other three cases the individual truth values of "p" and "q" could not have been determined. In such cases more information (statements) would have to be given to arrive at the exact values for "p" and "q." Let's illustrate this with the following problem:

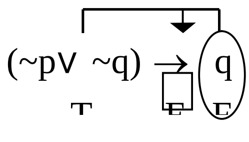
Problem: Determine the individual truth values of "p" and "q" given the following two statements:

(1): p ∧ ~q is " F"   (2): (~p ∨ ~ q) →q is " F."

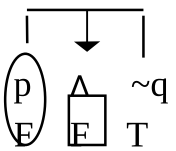
Solution:

| p | ∧ | ~ | q |
|---|---|---|---|
| T | F | F | T |
| T | **T** | T | F |
| F | F | F | T |
| F | F | T | F |

Since the first statement (p ∧ ~q is " F") is a false (F) conjunction, we don't have enough information to determine the individual values of "p" and "q." Constructing a truth table for "p ∧ ~q" shows that cases 1, 3 and 4 all have the same value (F).




The second statement [(~p ∨ ~ q) →q is " F" ], however, gives us a little more information. Since it's an implication, the only way it can have an "F" value is if the hypothesis is "T" and the conclusion (q) is "F." Thus, "q is F" which means "~q" must be "T."




Knowing that "~q" is "T," we have enough information to determine the value of "p" by returning to the first statement (p ∧ ~q is "F"). For a conjunction to be false at least one part of the conjunction must be false. Since "~q" is "T," then "p" must be "F." Therefore, we have found that "p" is "F" and "q" is "F." To make sure these values are correct they should be checked in both statements.

# Test 3

P. 97 Memorize Valid Form 1: **Law of Detachment** (or Modus Ponens)

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad\;\;} \\ \therefore \quad q \end{array}$$

(Problem set 1a on page 106 shows that it is a tautology: true in all four cases of the truth table.)

The first premise is a conditional statement; it guarantees getting "q" once the existence of "p" is established. The second premise affirms that we do have statement "p." Therefore, the conjunction of the two premises gives us "q." The two " p's" and the two "q's" match; each matching pair has been enclosed in an oval: the statements in the "p" positions (hypothesis side) must be the same and the statements in the "q" positions (conclusion side) must also be the same.

P. 98

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q \qquad\;\;} \\ \therefore \quad \sim p \end{array}$$

To show the validity of "Valid Form 2" we can construct a truth table, or we may simply compare this argument or an equivalent form of this argument to "Valid Form 1." We want to use "Valid Form 1" since this argument, like "Valid Form 1" has only two premises: one an implication (→) statement and the other a simple statement that cannot be converted to an implication statement. We now note that the statements on the hypothesis side ("p" and "~q") don't match, indicated with a dotted oval.

$$\begin{array}{l} \sim q \rightarrow \sim p \\ \underline{\sim q \qquad\qquad} \\ \therefore \quad \sim p \end{array}$$

Next, we use the contrapositive of "p → q" (~q → ~p), which is equivalent to "p → q." Since now both "~q's" match and also both "~p's" match, this argument is similar in form to "Valid Form 1." Therefore, this argument must be valid; and since it is equivalent to the original argument, the original argument must also be valid.

P. 99 Memorize Valid Form 3: **Transitive Law** (or Chain Law)

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

(A truth table would establish its validity -- see problem 1c on page 106.)
Note that both the premises and the conclusion are conditional statements. Statement "p" brings us to statement "q" and statement "q" brings us to statement "r"; which is the same as statement "p" bringing us to statement "r." Statement "q" can be thought of as the connecting link in the chain.

P. 100

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim p \qquad\;\;} \\ \therefore \quad \sim q \end{array}$$

To show that "Invalid Form 1" is invalid we first note that it contains two premises: one is an implication (→) statement and the other a simple statement. Therefore, we must compare this argument with Valid Form 1. Next we note that "p" and "~p" are not the same.

$$\begin{array}{l} \sim q \rightarrow \sim p \\ \underline{\sim p \qquad\qquad} \\ \therefore \quad \sim q \end{array}$$

The only possible change we can make to get this argument to look like "Valid Form 1" is to use the contrapositive (~q → ~p). Doing so, we see that "~q" and "~p" also don't match. This is sufficient for us to conclude that the argument is not valid.

P. 101

$$\begin{array}{l} p \rightarrow q \\ \underline{q \qquad\;\;} \\ \therefore \quad p \end{array} \;\; \equiv \;\; \begin{array}{l} \sim q \rightarrow \sim p \\ \underline{q \qquad\qquad} \\ \therefore \quad p \end{array}$$

To show that "Invalid Form 2" is invalid we note that the "p" and "q" don't match, and with the use of the contrapositive we note that "~q" and "q" also don't match. Since we cannot get this argument to look like "Valid Form 1," we may conclude that it is not valid.

Recall that this symbol means equivalent.

P. 102

$$\begin{array}{l} p \vee q \\ \underline{\sim p \qquad} \\ \therefore \quad q \end{array} \;\; = \;\; \begin{array}{l} \sim p \rightarrow q \\ \underline{\sim p \qquad\;\;} \\ \therefore \quad q \end{array}$$

To show the validity of "Valid Form 4" we may convert the disjunction (∨ ) to an implication (→) (as discussed on page 86) and note that the result is in the form of "Valid Form 1" (the **Law of Detachment**).

Another viewpoint: Since it is given that "p or q is true (T)," then at least "p" or "q" must be true (T). However, "p" is false (since premise "~p" is true). Therefore, "q" must be true (T), and the argument is then valid.

P. 103

| (original) | | (second) | | (third) |
|---|---|---|---|---|
| p ∨ q | | ~p → q | | ~q → p |
| ~q | ≡ | ~q | ≡ | ~q |
| ∴ p | | ∴ p | | ∴ p |

To show the validity of "Valid Form 5" we may first convert the disjunction (∨ ) to an implication (→). Since "~p" and "~q" don't match, we may try the contrapositive. Doing so, we find that the third argument is valid by the **Law of Detachment**. Since the original argument is equivalent to the second argument and the second argument is equivalent to the third argument, which is valid, then the original argument must also be valid.

P. 104

| | | | | |
|---|---|---|---|---|
| p ∨ q | | ~p → q | | ~q → p |
| p | ≡ | p | ≡ | p |
| ∴ ~q | | ∴ ~q | | ∴ ~q |

To show that "Invalid Form 3" is invalid we first convert the disjunction (∨ ) to an implication (→). Since "~p" and "p" don't match, we may try the contrapositive. Since "~q" and "p" also don't match, the original argument is not valid.

Another viewpoint: Since it is given that "p or q is true (T)," then either "p" is true (T) or "q" is true (T) or possibly both are true (T). Since premise "p" is also given to be true (T), "q" could either be true (T) or false (F). The conclusion "~q" (assumed to be true (T)) only provides for "q" to be false (F). Therefore, the argument is not valid.

| | | | | |
|---|---|---|---|---|
| p ∨ q | | ~p → q | | ~q → p |
| q | ≡ | q | ≡ | q |
| ∴ ~p | | ∴ ~p | | ∴ ~p |

To show that "Invalid Form 4" is invalid we may first convert the disjunction (∨ ) to an implication (→). Since "~p" and "q" don't match, we try the contrapositive. Since "~q" and "q" also don't match, the original argument is not valid.

NOTE: The **Law of Detachment** and the **Transitive Law** are the only two valid forms you need to know. Also, you don't need to memorize any of the invalid forms. If it is impossible to convert the argument to obtain one of the two valid forms, then it is invalid. In order to convert an argument you must substitute (or interchange) one or more of the three logically equivalent statements given above (P.86 - P.88), and possibly one of **DeMorgan's Laws** (P.61/P.65). For example, to show that the argument on the right is valid, consider the following sequence of five steps (or substitutions).

| |
|---|
| ~(~p ∧ ~q) |
| ~q |
| ∴ p |

① DeMorgan's Law ③ Contrapostive ④↓ Law of Detachment

| | | | | | | |
|---|---|---|---|---|---|---|
| ~(~p∧ ~q) | | p∨ q | | ~p → q | | ~q → p |
| ~q | ≡ | ~q | ≡ | ~q | ≡ | ~q |
| ∴ p | | ∴ p | | ∴ p | | ∴ p |

② Disjunction to Implication

⑤ Therefore, the first argument is equivalent to the last argument, which is valid by the **Law of Detachment**; hence, the original argument is valid.

NOTE: The method of Substitution is also helpful when we are asked to supply a conclusion to make an argument valid.

| |
|---|
| p → q |
| q → r |
| ∴ r → p |

Let's determine if the argument on the left is valid. Note that it is similar to "Valid Form 3" (**The Transitive Law**) since it has two implication (→) statements for it's premises. However, instead of obtaining "p→r," we have the converse (r → p). Since the converse is not logically equivalent to the implication, the argument is invalid. Note that substituting one logically equivalent statement for another was not needed in this example.

115 # 4.

Let p be: A person has knowledge of music.

Let q be: A person enjoys Mozart.

$p \to q$ (premise 1)

and

Let r be: A person has knowledge of theater.

Let s be: A person enjoys Sondheim.

$r \to s$ (premise 2)

Recall that the conjunction ($\wedge$) of all premises is the hypothesis of an argument. Also, the rearrangement of statements does not affect the truth value of a conjunction.

and

Kathy enjoys Mozart and Kathy does not enjoy Sondheim is: $q \wedge \sim s$ (premise 3)

$q$ and $\sim s$

Separating the third premise ($q \wedge \sim s$) into two component statements and then rearranging them gives us the following two arguments shown below.

$p \to q$
$q$
$\therefore$ ☐

Since the premises of this argument cannot be put in the form of <u>Valid Form 1</u>, we cannot reach a conclusion from this part.

and

$r \to s$
$\sim s$
$\therefore$ ☐

Here, however, we can reach a conclusion with the help of the contrapositive:

$\sim s \to \sim r$
$\sim s$
$\therefore \sim r$: Kathy does not have knowledge of theater.

<u>P. 116</u> **<u>Euler Circles</u>**: Your strategy should be to make as many diagrams as possible that represent the premises. <u>Each</u> diagram must support the stated conclusion if the argument is to be <u>valid</u>. If you can find <u>one</u> diagram that supports the premises but <u>fails</u> to yield the stated conclusion, the argument is <u>not valid</u>.

<u>P. 116</u> <u>Problem 1</u>: Since there is only <u>one</u> diagram that can be drawn which represents the premises and supports the conclusion, then that one diagram is the <u>significant diagram</u>.

<u>P. 117</u> <u>Problem 2</u>: Since there are <u>two</u> diagrams that can be drawn to represent the premises, but the second one fails to support the conclusion, the argument is <u>not valid</u>. The significant diagram is the one that <u>fails</u> to support the conclusion -- the <u>second</u> one.

7

<u>P. 119</u> <u>Problem 3</u>: Since the argument is <u>not valid</u>, a significant diagram is <u>either</u> the first <u>or</u> the second one; both diagrams represent the premises but fail to support the conclusion.

<u>P. 121</u> <u>Problem 4</u>: The significant diagram is the one that represents the premises but fails to support the conclusion (shown on page 123).

<u>P. 123</u> <u>Problem 5</u>: Since there is only <u>one</u> diagram that represents the premises and also supports the conclusion, it is the significant diagram.

<u>P. 124</u> <u>Problem 6</u>: Since the argument is <u>not valid</u>, a significant diagram is <u>either</u> the first <u>or</u> the second one. Both diagrams represent the premises but fail to support the conclusion. Note the final paragraph on page 124.

<u>P. 126</u> <u>Problem 7</u>: Since the first diagram does <u>not</u> support the <u>entire</u> conclusion, then the argument is <u>not valid</u> and the <u>first</u> diagram is the significant one.

<u>P. 128</u> <u>Problem 8</u>: Since <u>both</u> diagrams represent the premises and each supports the conclusion, then the argument is <u>valid</u> and <u>both</u> diagrams are significant.

<u>NOTE</u>: Testing the validity of an argument using **Euler Circles** is similar in strategy to testing validity using truth tables. With truth tables, to show validity we must <u>show all</u> cases are true (T); with **Euler Circles**, we must <u>show all</u> possible diagrams that represent the premises also support the conclusion. With truth tables, to show that an argument is <u>not</u> valid, we must <u>show</u> there is at least <u>one</u> case that is false (F); with **Euler Circles**, we must <u>show</u> at least <u>one</u> diagram represents the premises but fails to support the conclusion.