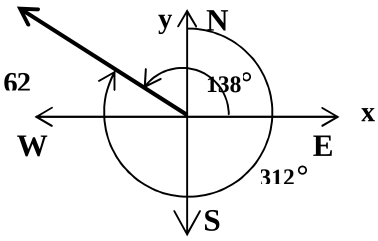
More on Vectors

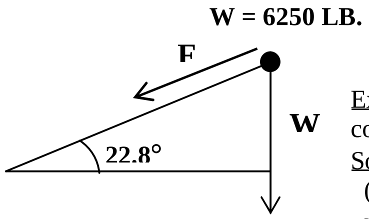
I. Resolving a vector into its horizontal and vertical components:

In section 2.5, you learned how to find the magnitudes of the horizontal and vertical vectors, $|V_x|$ and $|V_y|$, respectively, of any given vector. The vectors V_x and V_y, called, respectively, the horizontal and vertical components of V, are determined as follows: $V_x = |V| \cos\theta$, $V_y = |V| \sin\theta$ where θ is the standard - position (not reference) angle of V. (Note: The x- and y- components in this handout are rounded for the sake of illustration only. When you do problems yourself, you should be careful to keep all calculator digits until rounding the final answer.)

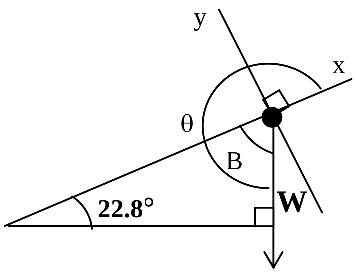
Example 1: Find the horizontal and vertical components of a vector V with the magnitude 626 and standard position angle 212° .
Solution: $V_x = 626 \cos 212° = -531$, $V_y = 626 \sin 212° = -332$. Note that the negative value for V_x and V_y reflect the fact that the x- and y- components are directed in the negative directions of the x- and y- axes respectively. Finding these values is called "resolving V into its x- and y- components." Note also that no diagram need be drawn to solve such a problem.



Example 2: A wind blows at 62.0 miles per hour at a heading of 312°. Find its components.
Solution: If we consider our compass directions to be aligned along the x- and y- axes in the traditional manner (see left), we first note that a heading of 312 ° corresponds to a standard-position angle of 138 °. Thus, $V_x = 62.0 \cos 138° = -46.1$, $V_y = 62.0 \sin 138° = 41.5$. Interpreting these results in terms of compass directions, we find that the wind has a component of 46.1 miles per hour towards the west and a component of 41.5 miles per hour towards the north.



Example 3: A 6250 lb. truck is parked on a ramp inclined 22.8° with the horizontal. Find the component of its weight directed down the ramp.
Solution: The physical situation illustrated at left where the 6250 lb. weight is directed down (toward the center of the earth) and the force F is to be determined. One way to solve the problem is to set up our rectangular coordinate system so that F will be parallel to one of the axes as shown in the second diagram. Since B is complementary to 22.8 °, B= 67.2 °, and the standard-position angle θ of the weight vector in our coordinate system is θ = 180 ° + B = 247.2 °. We have set up our coordinate system so that $F = W_x = |W| \cos\theta = 6250 \cos 247.2° = -2422$ lbs, where the negative simply indicates the negative x- direction in our coordinate system. Thus, the answer is 2422 lbs.



II. Adding vectors:

In section 2.5, you found the magnitude and the reference angle, θ_{ref}, of the direction of a vector from its x- and y- components as follows:

$$|V| = \sqrt{|V_x|^2 + |V_y|^2} \qquad \tan\theta_{ref} = \frac{|V_y|}{|V_x|}$$

We can find the standard - position angle θ of the vector if we use the above in connection with the signs of V_x and V_y.

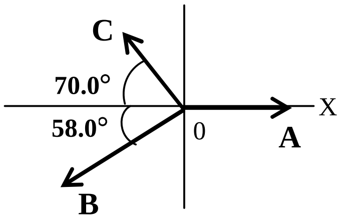
Example 4: If a plane heads due west at 210 miles per hour and if a 50 mile per hour wind blows from due south, find the actual velocity of the plane with respect to the ground. The velocity V_p imparted to the plane by the plane and the velocity V_w imparted to the plane by the wind are the x- and y- components of its resulting velocity V with respect to the ground. If we align the rectangular coordinate system and the compass directions in the usual way (as in Example 2) we have $V_x = -210$ (since due west is the negative x- direction) and $V_y = 50$ (since the wind is towards due north).

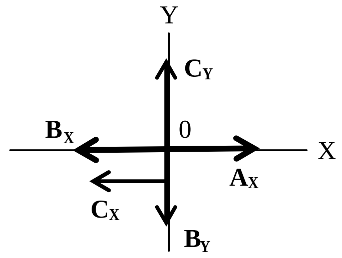
Thus, $|V| = \sqrt{|V_x|^2 + |V_y|^2} = \sqrt{210^2 + 50^2} = 216$; and , $\tan\theta_{ref} = \frac{|V_y|}{|V_x|} = \frac{50}{210}$ giving $\theta_{ref} = 13.4°$. Since $V_x < 0$ and $V_Y > 0$, θ is a second quadrant angle. Therefore, the velocity of the plane is 216 miles per hour, 13.4 ° N of W and, hence, at a heading of 283.4 °.

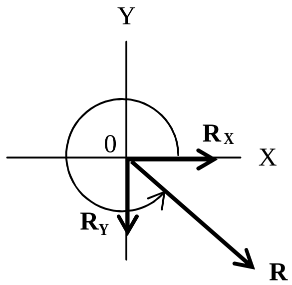
Finally, we can combine the problems of resolving vectors into components and recombining components, as in the previous example, to add any two or more vectors. The sum is called the resultant.

Example 5: Find the resultant of the three vectors A, B, C shown below with |A| = 444, |B| = 388, and |C| = 222.

Solution:







First we resolve each of the three vectors into their x- and y- components, being careful to use the appropriate standard -position angles. Then, since the x- and y- components are in the same or opposite directions, we can add them directly to find R_X and R_y.

Vector	x-component	y-component
A	444 cos 0 ° = +444	444 sin 0 ° = 0
B	388 cos 238° = -206	388 sin 238 ° = -329
C	222 cos ll0 ° = -76	222 sin 110° = +209
R	+162	-120

Finally, we obtain

$$|R| = \sqrt{162^2 + 120^2} = 202; \text{ and } \tan\theta_{ref} = \frac{120}{162}, \theta_{ref} = 36.5°(\text{QIV}), \theta = 323.5° \text{ (the standard-position angle)}$$

Example 6: A plane travels 550 miles per hour at a heading of 345 ° and then 842 miles at a heading of 20°. Find its displacement from its starting point.

Solution: Once we realize that we are finding the resultant of two displacement vectors, there is no need to draw a diagram, except possibly to convert the headings of 345 ° and 20 ° to their corresponding standard-position angles of 105° and 70° respectively.

Vector	x-component	y-component
A	550 cos 105° = - 142	550 sin 105° = +531
B	842 cos 70° = + 288	842 sin 70° = +791
R	+146	+1322

Finally, we obtain

$$|R| = \sqrt{|R_x|^2 + |R_y|^2} = \sqrt{146^2 + 1322^2} = 1330; \text{ and } \tan\theta_{ref} = \frac{|R_y|}{|R_x|} = \frac{1322}{146}, \theta_{ref} = 83.7° \quad \theta = 83.7°$$

Therefore, the plane is 1330 miles from its starting point at a heading of 6.3 ° (corresponding to the standard-position angle of 83.7°).

Example 7: Find the resultants of the forces F and G given below in terms of magnitudes and standard-positions angles: $|F| = 543$ lbs, $\theta_F = 173°$

$|G| = 445$ lbs, $\theta_G = 207°$

Solution:

$R_x = F_x + G_x = 543 \cos 173° + 445 \cos 207° = -935$

$R_Y = F_Y + G_Y = 543 \sin 173° + 445 \sin 207° = -136$

$$|R| = \sqrt{|R_x|^2 = |R_y|^2} = \sqrt{935^2 + 136^2} = 945 \text{ lbs; and } \tan q_{ref} = \frac{|R_y|}{|R_x|} = \frac{136}{935}, \quad q_{ref} = 8.3°, \quad q = 188.3°$$

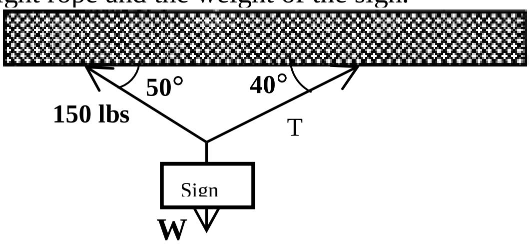
Example 8: If the two forces in Example 7 act on an object, what third force is needed to keep it from moving?

Solution: When forces act on an object and it remains motionless, we say that the forces are in equilibrium; that is, their sum is the vector with magnitude zero. Since we already have added the forces F and G in Example 7 to find their resultant, the third force must be equal in magnitude and opposite in direction for the sum of all three to be zero. Therefore, the answer is 945 lb. with a direction of 8.3 °.

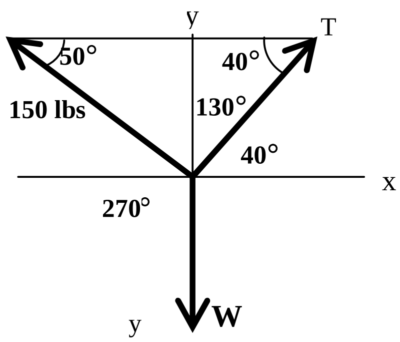
If we had not found the resultant of F and G already, we would start by finding the sum of their x- and y- components to be -935 and -136 respectively (as we did in Example 7.) Then the x- and y- components of the third force would be their opposites +935 and +136, respectively, for their totals to be 0. Finally, we would find the third force from these components as usual.

Often a problem involving vectors may be solved through more than one approach. The following is such a problem:

Example 9: A sign hangs from two ropes as shown below. If the tension in the left rope is 150 lbs, find the tension in the right rope and the weight of the sign.



Solution 1:



Solution 1: Since the sign is in equilibrium, the sum of the horizontal components must be zero; that is:

$150 \cos 130° + |T| \cos 40° + |W| \cos 270° = 0$

$|T| \cos 40° = -150 \cos 130°$

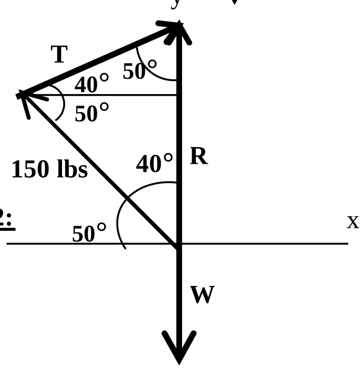
$$|T| = \frac{-150 \cos 130°}{\cos 40°} = 126 \text{ lbs}$$

Also, the sum of the vertical components must be zero:

$150 \sin 130° + 126 \sin 40° + |W| \sin 270° = 0$

$$|W| = \frac{-150 \sin 130° - 126 \sin 40°}{\sin 270°} = 196 \text{ lbs.}$$

Solution 2:



Solution 2: We add vector T to the 150 lb. tension vector geometrically as in the third diagram, forming a triangle. Since the sign is in equilibrium, the resultant R must be equal to W in magnitude, that is, $|R| = |W|$. Since the interior angle formed at the origin must be complimentary to 50°, it equals 40 °; and thus, the remaining interior angle , formed by T and R must be 50°. (Why?) We now have enough information to use the Law of Sines to find $|T|$ and $|W| = |R|$:

$$\frac{|T|}{\sin 40°} = \frac{|W|}{\sin 90°} = \frac{150}{\sin 50°}, \quad |T| = 126 \text{ lbs. and } |W| = 196 \text{ lbs.}$$

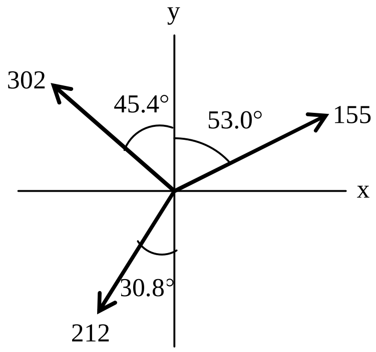
Exercises:

1. Find the resultant of the following vectors:

a) $A = 56.0, \theta_A = 76.0°$
   $B = 24.0, \theta_B = 200.0°$

b) $A = 21.9, \theta_A = 236.2°$
   $B = 96.7, \theta_B = 11.5°$
   $C = 62.9, \theta_C = 143.4°$

c)

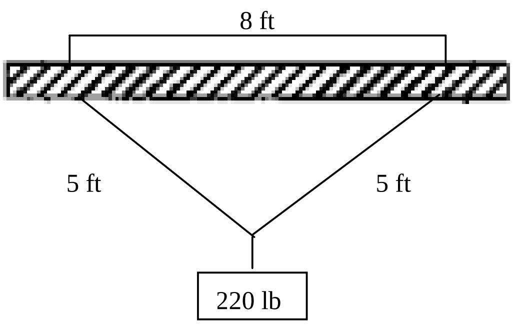


2. A 100 lb block rests on an incline plane which makes an angle of 20° with the horizontal. Find the force perpendicular to the plane.

3. A ship sails 15.0 km on a heading of 200°, then 30.0 km on a heading of 350° . How far and at what heading must it travel to return to port?

4. A 120 lb gymnast hangs by her hands from a high horizontal bar. Her hands are separated so that her hands form 65° angles with the bar. Find the tension in her arms. (Note: Each arm supports half her weight.)

5. Find the tension in the ropes supporting the 220 lb sign below.



6. A boat which travels 9.00 miles per hour in still water aims 50.0 ° upstream against a current running 4.00 miles per hour. Find its resultant velocity.

Answers:

1. a) $R = 47.0, \theta = 101.0°$
   b) $R = 50.2, \theta = 50.3°$
   c) $R = 235, \theta = 148.3°$

2. 94.0 lbs

3. 18.6 km at 146.2 °

4. 66.2 lbs

5. 183 lbs

6. 6.47 miles per hour, 26.6 ° upstream

MATH 1210 TEST #6

SOLUTIONS TO VECTOR HANDOUT

1A

$\vec{A}$	56 at 76°	$A_x \approx 13.55$	$A_Y \approx 54.34$
$\vec{B}$	24 at 200°	$B_x \approx -22.55$	$B_Y \approx -8.21$
$\vec{R}$		$R_x \approx -9$	$R_Y \approx 46.13$

$$R =, \sqrt{(-9)^2 + (46.13)^2} \approx 47.0$$

$$\theta_{ref} = \tan^{-1}\left(\frac{46.13}{9}\right) \approx 79.0°, \theta \text{ in Quad II}$$

$$\therefore \theta = 180-79 = 101°$$

1B

$\vec{A}$ 21.9 at 236.2°

$\vec{B}$ 96.7 at 11.5°

$\vec{C}$ 62.9 at 143.4°

$\vec{R}$

Component

X	Y
-12.2	-18.2
94.8	19.3
-50.5	37.5
32.1	38.6

$$R = \sqrt{(32.1)^2 + (38.6)^2} \approx 50.2$$

$$\theta_{ref} = \tan^{-1}\left(\frac{38.6}{32.1}\right) \approx 50.3°, \text{ in Quad I}$$

$$\therefore \theta = 50.3°$$

1C

$\vec{A}$ 155 at 37°

$\vec{B}$ 302 at 135.4°

$\vec{C}$ 212 at 239.2°

$\vec{R}$

Component

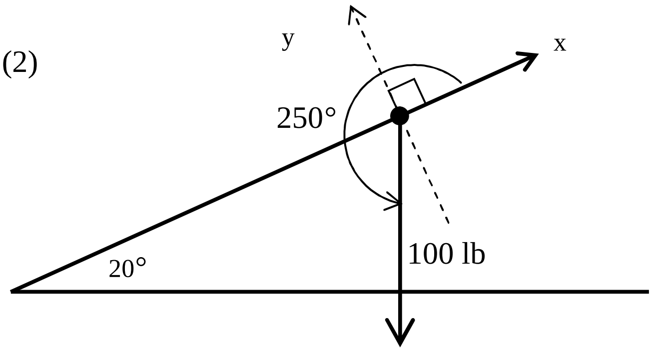
X	Y
123.8	93.3
-215.0	212.1
-108.6	-182.1
-199.8	123.3 (2nd Quad)

$$R = \sqrt{(-199.8)^2 + (123.3)^2} \approx 234.8$$

$$\theta_{ref} = \tan^{-1}\left(\frac{123.3}{199.8}\right) \approx 31.7°, \theta \text{ in Quad II}$$

$$\therefore \theta = 180° - 31.7 = 148.3°$$

(2)

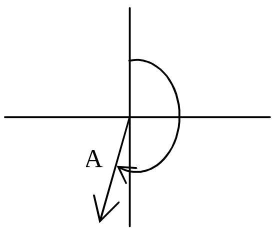


$F_Y = 100 \text{ lb} \sin 250°$

$= -94 \text{ lb}$

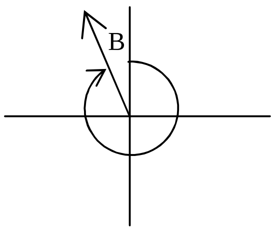
$|F_Y| = 94 \text{ lb}$

(3) <u>By Components:</u>



15 km heading 200°

Using standard position angle of 250°



30 km heading 350°

Using stand position angle of 100

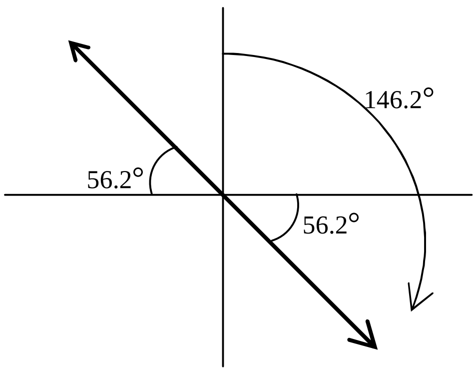
A_x	A_y
15 cos 250°	15 sin 250°
-5.13	-14.10

B_x	B_y
30 cos 100°	30 sin 100°
-5.21	29.54

resultant	R_x	R_y
(adding)	-10.34	15.44 (2nd Quad)

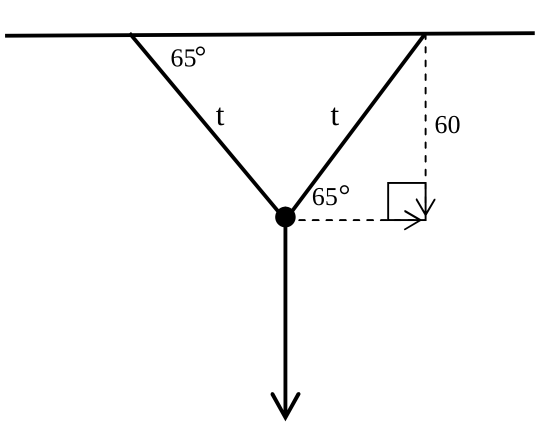
$$\vec{R} = \sqrt{(-10.34)^2 + (15.44)^2} = \boxed{18.6}$$

$$\theta_{ref} = \tan^{-1}\left(\frac{15.44}{10.34}\right) \approx 56.2°$$



Since R is in QII, the angle to <u>return</u> to port would be directly opposite, with a reference angle of 56.2° in QIV, that is, a heading of 90° + 56.2° = 146.2°.

(4)



$$\sin 65° = \frac{60}{t}$$

$$t = \frac{60}{\sin 65°} \approx 66.2$$

$$t \approx 66.2$$

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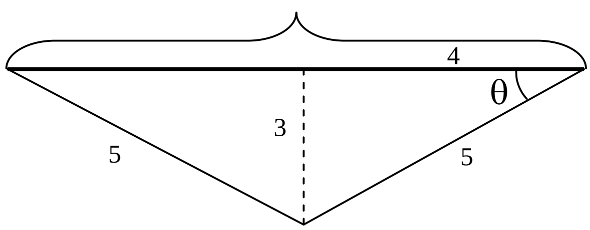
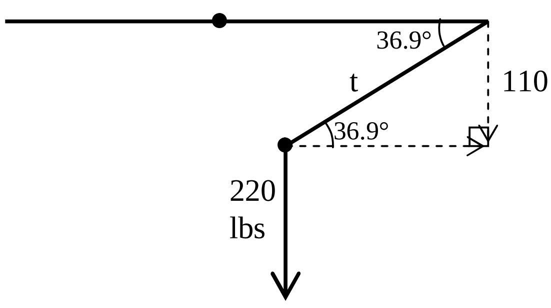
(5)



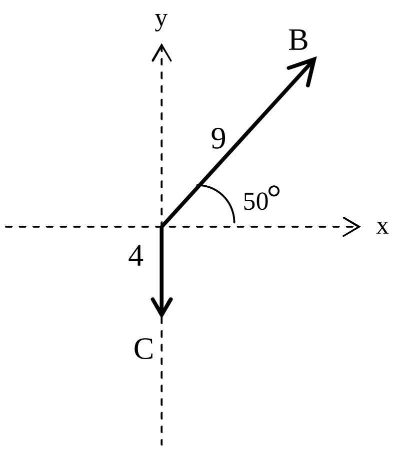
tan θ = ¾ , θ ≈ 36.9°
following exercise 4



$$\sin 36.9° = \frac{110}{t}$$

$$t = \frac{110}{\sin 36.9°} \approx 183$$

(6) <u>By Components:</u>



B_x = 9cos 50° = 5.785	B_y = 9 sin 50° = 6.894
C_x = 4 cos 270° = 0	C_y = 4 sin 270° = - 4.000
R_x = 5.785	R_y = 2.894

$$R = \sqrt{R_x{}^2 + R_y{}^2} = 6.47 \text{ mph}$$

$$\text{IV} \quad \tan\theta_R = \frac{R_y}{R_x} = \frac{2.894}{5.785} = .5003$$

$$\Rightarrow \quad \theta_{Rref} = 26.6°, \quad \theta \text{ in Quad I}$$

$$\therefore \quad \theta_R = 26.6°$$