LABELING SUPPLEMENT

Revised 1/5/04

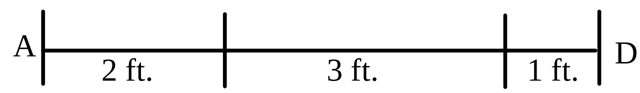
When asked to determine the area of a plane figure or the volume of a solid, we often get mixed up by the labels. These are the three types of labels with which we will be concerned:

A. length or distance (linear measure)	Ex. feet, inches, yards, etc.
B. area (square measure)	Ex. square feet, square inches, square centimeters, etc.
C. volume (cubic measure)	Ex. cubic feet, cubic inches, cubic meters, etc.

A. Let's begin with linear measure. It's just as it seems, distance on a line.

Suppose we ask the following:

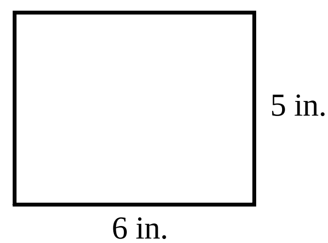
Given the figure to the right, what is the distance from A to D?



You would answer 6 feet (6ft.).

What about the figure on the right? If you are asked to find the perimeter, you are being asked for the distance along the edge of the rectangle. In this case, it is the sum of the four sides and equals 22 inches (22in.). Use the formula for the perimeter of a rectangle

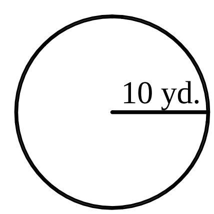
$P = 2L + 2W = 2 \times 6 \text{ in.} + 2 \times 5 \text{ in.} = 22 \text{ in.}$



Let's look at another example of linear measure. What is the distance around the circle shown here? Recall that the perimeter of a circle has a special name, circumference.

For this problem we have the formula: $c = 2 \times \pi \times r$

π is represented by either 3.14 or 22/7, whichever you prefer, unless a particular value is listed in the problem. r is the radius of the circle.



$$c = 2 \times \pi \times r$$
$$= 2 \times 3.14 \times 10 \text{ yd.}$$
$$= 62.8 \text{ yards}$$

or

$$c = 2 \times \pi \times r$$
$$= 2 \times \frac{2}{7} \times 10 \text{ yd.}$$
$$= \frac{440}{7} \text{ yards or } 62\frac{6}{7} \text{ yards}$$

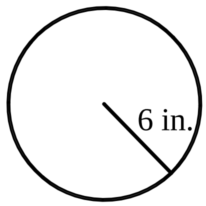
The circumference could also be determined by using the formula $c = \pi \times d$ where d is the diameter of the circle. The diameter is twice the length of the radius. So, in this example the diameter is $2 \times 10 \text{ yd.} = 20 \text{ yd.}$ Therefore,

$$c = \pi \times 20 \text{ yd.}$$
$$= 3.14 \times 20 \text{ yd.}$$
$$= 62.8 \text{ yd.}$$

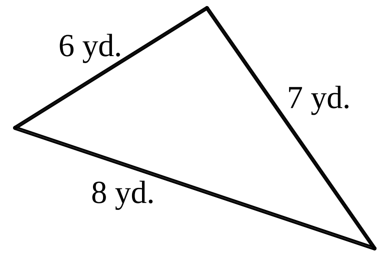
Similarly, if we let $\pi = \frac{22}{7}$ we would get $c = \frac{440}{7}\text{yd.}$ or $62\frac{6}{7}$ yd.

Exercise I Find the perimeter. (Answers are at the end of the handout.)

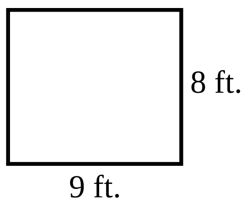
A.



B.



C.



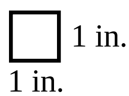
D.

Be careful of this one!

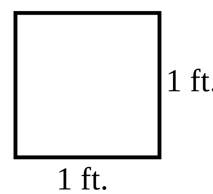
B Our next labeling difficulty is area.

When we ask for area, we are asking how many squares it will take to cover this figure. Therefore, we will use square feet, square inches, or some other square unit. A square unit is a four-sided figure with parallel and equal sides?

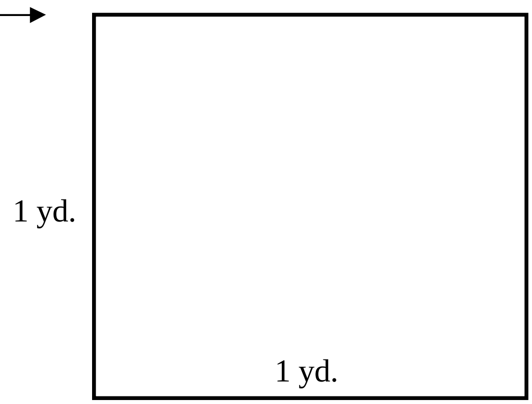
A square inch →



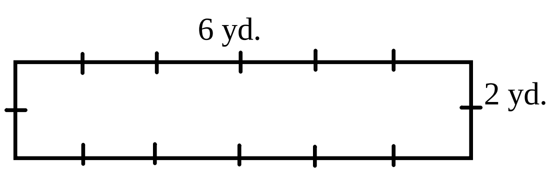
A square foot →



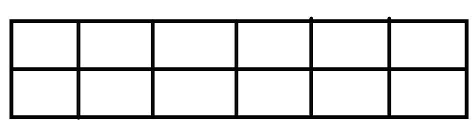
A square yard →



Take a look at the following figure:



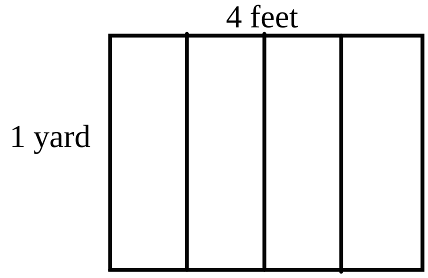
If we mark the units along the length and width, we have the following:

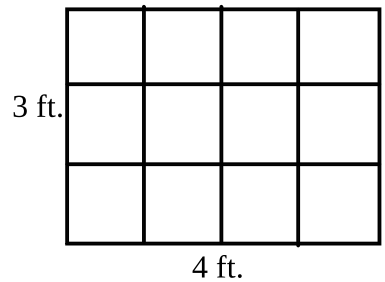


As you can see, we have 12 squares in our latest figure and each square is 1 yard by 1 yard, or we can say that the figure contains 12 square yards. Its area is 12 sq. yd. which can also be written 12 yd^2.

We can get the area by multiplying the length (6 yards) by the width (2 yards) to get the number 12. Our label, square yards, comes from knowing that we are measuring area.

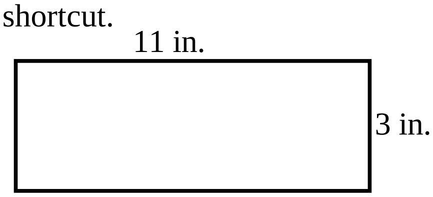
Let's try another example.
In this case we don't get squares. We get figures that are I foot by I yard. It is necessary then to make sure the sides are expressed in the same unit, that is, both feet or both yards. (Recall 1 yd. = 3 ft.)
Let's start this one again and change all units to feet. We can see that we have12 squares. Therefore, the area is 12 square feet (12 ft^2). Again, we could have multiplied the length (L) by the width (W) to get 12 square feet.





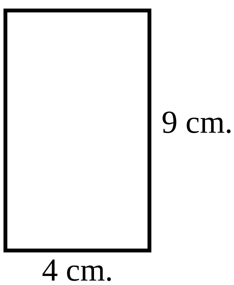
Let's show a few more examples using the formula as a shortcut.

A.



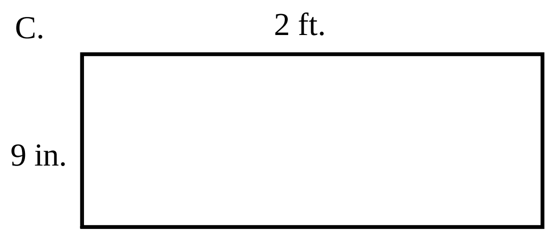
$A = L \times W$
$= 11 \text{ in.} \times 3 \text{ in.}$
$= 33 \text{ in}^2$ or 33 sq. in.

B.



$A = L \times W$
$= 9 \text{ cm.} \times 4 \text{ cm.}$
$= 36 \text{ cm}^2$ or 36 sq. cm.

C.



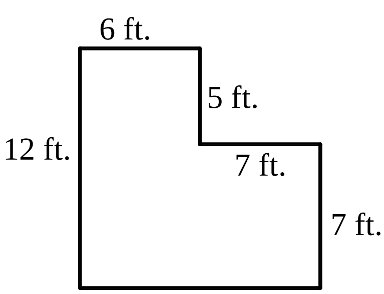
$A = L \times W$

$= 24 \text{ in.} \times 9 \text{ in.}$

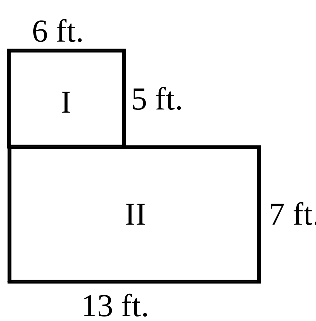
$= 216 \text{ in}^2$ or 216 sq. in.

To make sure the units are the same, we changed 2 feet to 24 inches.

Suppose we have a figure that is not a rectangle, say a figure like this:



What we will have to do is divide it into sections we can handle, the rectangles shown below.

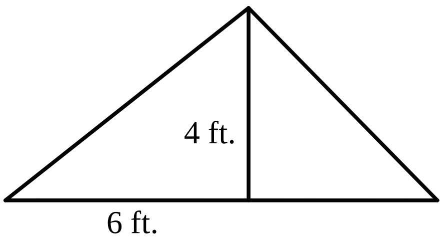


The area of rectangle I is:
$A = 5 \text{ ft.} \times 6 \text{ ft.}$
$= 30 \text{ ft}^2$

The area of rectangle II is:
$A = 13 \text{ ft.} \times 7 \text{ ft.} = 91 \text{ ft}^2$

Therefore, the total area is $30 \text{ ft}^2 + 91 \text{ ft}^2 = 121 \text{ ft}^2$

What if we had a triangle instead?

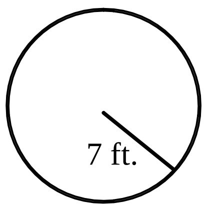


We would use the formula $A = \frac{1}{2} \times b \times h$ where b is the base (6 ft.) and h is the height (4 ft.).

$A = \frac{1}{2} \times 6 \text{ ft.} \times 4 \text{ ft.} = 12 \text{ ft}^2$

This means that we could take 12 squares, each 1 foot by 1 foot and if we cut them correctly, they would exactly fill this triangle.

Now let's find the area of a circle. We will use the formula $A = \pi \times r^2 = \pi \times r \times r$.

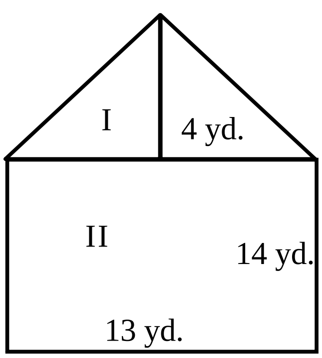


Therefore, $A = 3.14 \times 7 \text{ ft.} \times 7 \text{ ft.}$ or $A = \frac{22}{7} \times 7 \text{ ft.} \times 7 \text{ ft.}$

$= 153.86 \text{ ft}^2$ $\qquad$ $= 154 \text{ ft}^2$

This means we could fit 153.86 (or 154) squares into this circle.

Here are a few more examples. Find the area.



$A_I = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 13 \text{ yd.} \times 4 \text{ yd.}$

$= 26 \text{ yd}^2$

$A_{II} = L \times W$

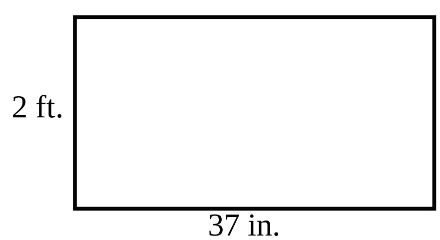
$= 13 \text{ yd.} \times 14 \text{ yd.}$

$= 182 \text{ yd}^2$

The total area is $26 + 182 = 208 \text{ yd}^2$

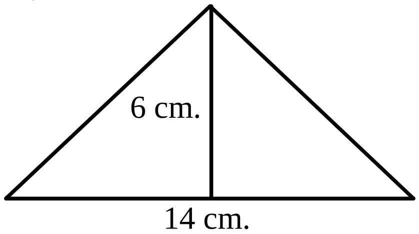
Notice that the next rectangle uses different units of measure.
We will have to change them both to inches. (2 ft. = 24 in.)

So we get
$A = L \times W$
$= 37 \text{ in.} \times 24 \text{ in.}$
$= 888 \text{ in}^2$

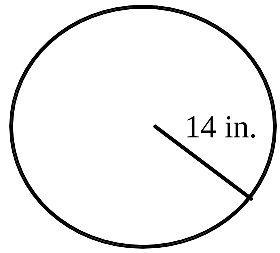


Exercise II Find the areas of the following figures. (Answers are at the end of the handout.)

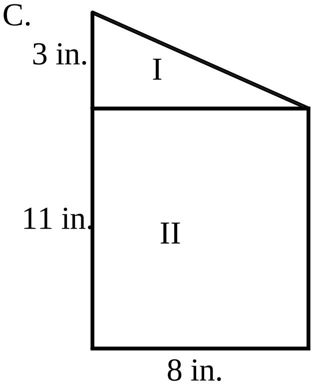
A.



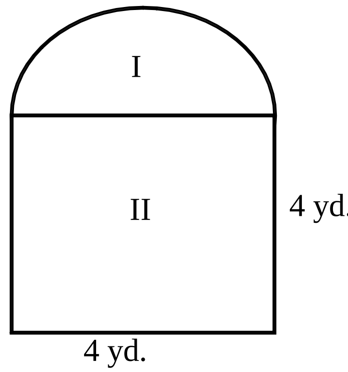
B.



C.

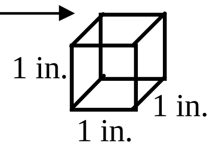


D.

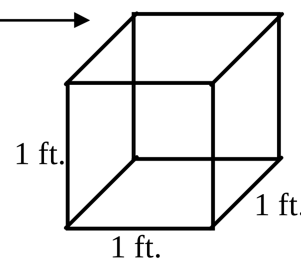


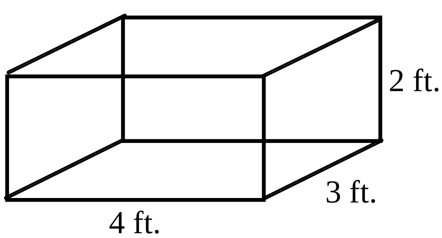
C. The final measure we will discuss is volume (or capacity).
When we ask for volume, we are asking how many cubes it will take to fill a solid.
Therefore, we will use cubic inches, cubic meters, or some other cubic unit.

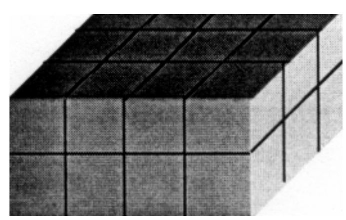
A cubic inch →



A cubic foot →





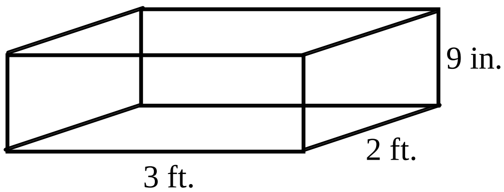


If we fill with cubes that are each 1 foot by 1 foot, as seen in the diagram to the right, it would hold 24 cubes. So, we would say that the volume is 24 cubic feet. This can be written as 24 cu. ft. or 24 ft^3.

We can get the volume of this type of figure, called a rectangular solid, by multiplying the length (L) times the width (W) times the height (H).

The formula then is $V = L \times W \times H$ and the answer is labeled in cubic units.

The next figure is another solid. Two of the dimensions are in feet and the other is in inches. We have to be sure to change them all to the same unit. (Recall 12 in. = 1 ft.)



So, 2 ft. = 24 in. and 3 ft. = 36 in.

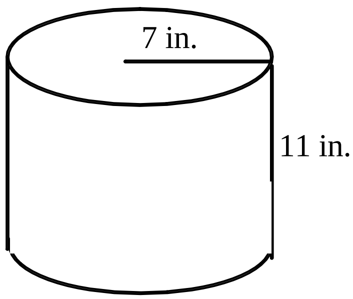
$V = L \times W \times H$

$= 36 \text{ in.} \times 24 \text{ in.} \times 9 \text{ in.}$

$= 7776 \text{ in}^3$.

To find the volume of a cylinder, we take the area of the base (a circle) and multiply it be the height. the area of a circle is given by the formula $A = \pi \times r^2$.

The formula for volume of a cylinder then is $V = \pi \times r^2 \times h$ where r is the radius of the circle that forms its top and its bottom and h is the height of the figure.



$V = \pi \times r \times r \times h$

$= \frac{22}{7} \times 7 \text{ in.} \times 7 \text{ in.} \times 11 \text{ in.}$

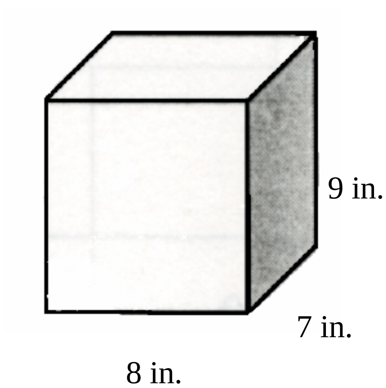
$= 1694 \text{ in}^3$ or

$V = 3.14 \times 7 \text{ in.} \times 7 \text{ in.} \times 11 \text{ in.}$

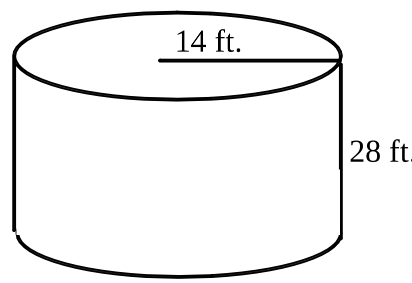
$= 1692.46 \text{ in}^3$

Exercise III Find the volume of each of the following figures. (Answers are at the end of the handout).

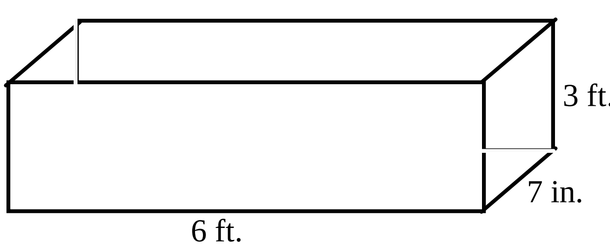
A.



B.



C.



ANSWERS TO EXERCISES

Exercise I

A.
$C = 2 \times \pi \times r$
$= 2 \times 3.14 \times 6$ in.
$= 37.68$ in. or
$C = 2 \times \frac{22}{7} \times 6$ in.
$= 37\frac{5}{7}$ in.

B.
$P = a + b + c$
$= 6$ yd. $+ 7$ yd. $+ 8$ yd.
$= 21$ yd.

C.
$P = 2L + 2W$
$= 2 \times 9$ ft. $+ 2 \times 8$ ft.
$= 18$ ft. $+ 16$ ft.
$= 34$ ft.

D. $P = 2L + 2W = 2 \times 24$ in. $+ 2 \times 4$ in. $= 48$ in. $+ 8$ in. $= 56$ in.
Note that 2 feet = 24 inches. We must express the length and the width in the same unit of measure.

Exercise II

A.
$A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 14$ cm. $\times 6$ cm.
$= 42$ cm^2

B.
$A = \pi \times r^2$
$= 3.14 \times 14$ in. $\times 14$ in. or
$= 615.44$ in^2

$A = \frac{22}{7} \times 14$ in. $\times 14$ in
$= 616$ in^2

C.
$A_I = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 8$ in. $\times 3$ in.
$= 12$ in^2
$A_{II} = L \times W$
$= 8$ in. $\times 11$ in.
$= 88$ in^2
So $A = A_I + A_{II}$
$= 12$ in^2 $+ 88$ in^2
$= 100$ in^2

D.
$A_I = \frac{1}{2} \times \pi \times r^2$ ($\frac{1}{2}$ the area of a circle)
$= \frac{1}{2} \times 3.14 \times 2$ yd. $\times 2$ yd.
$= 6.28$ yd^2 or $6\frac{2}{7}$ yd^2 (if $\pi = \frac{22}{7}$)
$A_{II} = L \times W$
$= 4$ yd. $\times 4$ yd.
$= 16$ yd^2
So $A = A_I + A_{II}$
$= 6.28$ yd^2 $+ 16$ yd^2
$= 22.28$ yd^2 or $22\frac{2}{7}$ yd^2

Exercise III

A.
$V = L \times W \times H$
$= 8$ in.. $\times 7$ in. $\times 9$ in..
$= 504$ in^3

B.
$V = \pi \times r^2 \times h$
$= 3.14 \times 14$ ft. $\times 14$ ft. $\times 28$ ft.
$= 17232.32$ ft^3 or 17248 ft^3

C.
$V = L \times W \times H$
$= 72$ in. $\times 7$ in. $\times 36$ in.
$= 18144$ in^3