MATH 2990 ADVANCED ENGINEERING MATHEMATICS

4 Credits
Offered in Lecture Format
Prerequisite required (MATH 2910 with a grade of C or better)

Revised 04/19/93

SYLLABUS

I. FIRST ORDER DIFFERENTIAL EQUATIONS

A. Definitions and notation
B. The solution concept
   1. General solutions
   2. Particular solutions
   3. Initial conditions
   4. Implicitly defined solution curves
   5. Examples in which existence or uniqueness is lacking
C. Separable equations
   1. Method of solution
   2. Applications
D. Exact equations
E. Linear equations
F. Introduction to simple circuits
   1. Resistors, inductors, and capacitors
   2. Single loop equation
   3. Examples leading to first order, linear equations
G. Orthogonal trajectories

II. SECOND ORDER, LINEAR EQUATIONS, PART 1

A. Definitions and observations
   1. Second order, linear equation
   2. Homogeneous case
   3. General solutions require two arbitrary constants
B. Solutions to homogeneous systems
   1. Linearly independent functions
   2. General solutions of the form
      $C_1 Q_1 (x) + C_2 Q_2 (x)$
C. The homogeneous, constant coefficient case
   1. Characteristic equation
   2. The three cases
   3. Applying initial conditions
D. Two theorems about homogeneous equations
   1. Existence - uniqueness result
   2. Positive coefficients, in the constant coefficient case, implies that every solution approaches zero
E. Applications of homogeneous equations
   1. Damped harmonic oscillator
   2. RLC circuit with constant EMF
F. Solving non-homogeneous equations with constant coefficients using undetermined coefficients
G. Applications of non-homogeneous equations

1. Forced harmonic oscillator
2. RLC circuit with sinusoidal EMF

III. **SECOND ORDER, LINEAR EQUATIONS, PART 2**

A. Homogeneous equations with polynomial coefficients; ordinary and singular points
B. Power series solutions about ordinary points
C. The Euler - Cauchy equation
D. Solutions about regular singular points; the method of Frobenius
E. Some special functions
   1. The gamma function
   2. The Bessel function
   *3. Hypergeometric functions

IV. **LAPLACE TRANSFORMS**

A. Motivation
B. Definition and examples
*C. Conditions which insure the existence of a Laplace transform
D. Some properties of Laplace transforms
   1. Linear operator property
   2. Transforms of derivatives
   3. Shifting on the s-axis
   4. The derivative of a transform
   5. The integral of a transform
   6. The convolution property
E. The unit step function
   1. Writing discontinuous functions in terms of unit steps
   2. Transforms of discontinuous functions
F. Solving differential equations using Laplace transforms; use of tables

V. **LINEAR ALGEBRA, PART 1**

A. Extending vector concepts to TR^n
   1. Components
   2. Magnitude
   3. Zero vector
   4. Unit vector
   5. Equality of vectors
   6. Scalar multiplication
   7. Adding and subtracting
   8. The dot product
   9. Basis vectors $\underline{e}_1$, $\underline{e}_2$, ...$\underline{e}_n$
B. Vector spaces
   1. Definition
   2. Examples
   3. Linearly independent sets
   4. Definition of basis; dimension

VI. **LINEAR ALGEBRA, PART 2**

A. Matrices
   1. Entries
   2. Notation
   3. Row vectors
   4. Column vectors

        5. Matrix equality
        6. Square matrices; main diagonal
        7. Zero matrix
        8. Submatrices
    B. Matrix operations
        1. Scalar multiplication
        2. Adding and subtracting
        3. Properties
        4. Matrices form a vector space
    C. More about square matrices
        1. Transpose
        2. Symmetric and skew-symmetric matrices
        3. Upper and lower triangular matrices
        4. Diagonal matrices; identity matrices
    D. Matrix multiplication
        1. General method
        2. Properties
    E. Systems of algebraic equations
        1. Matrix form
        2. Augmented matrix
        3. Elementary row operations; solving systems
        4. The rank of matrix
        5. Theorem relating systems to rank
    F. The inverse of a square matrix
        1. Definition
        2. Properties
        3. Singular matrices
        4. Finding inverses using row operations
    G. The determinant of a square matrix
        1. Expansion by minors; cofactors
        2. Properties of determinants
        3. A inverse exists if and only if det (A) □ 0
        4. Finding inverses using cofactors
        5. Cramer's rule
        6. A $\underline{x}$ = $\underline{0}$ has non-zero solutions if and only if det (A) = 0
    H. Quadratic forms
        1. Definition of positive definite
        2. Sylvester's criterion

*VII. **LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS**
    A. Definition and notation
    B. Solving systems using eigenvalues

*VIII. **INTRODUCTION TO PARTIA DIFFERENTIAL EQUATIONS**
    A. Heat equation
    B. Fourier series

*Optional