Name Date

Week: _____ **Problem:** _____

Given:

The angle θ measures the direction of the unit vector $\mathbf{e}$ relative to the x axis. The angle θ is given as a function of time by $\theta = 2t^2$ rad. What is the vector $d\mathbf{e}/dt$ at $t = 4$ s?

Required:

Solution:

Solution: By definition:

$$\frac{d\mathbf{e}}{dt} = \left(\frac{d\theta}{dt}\right)\mathbf{n},$$

where

$$\mathbf{n} = \mathbf{i}\cos\left(\theta + \frac{\pi}{2}\right) + \mathbf{j}\sin\left(\theta + \frac{\pi}{2}\right)$$

is a unit vector in the direction of positive θ. The angular rate of change is

$$\left[\frac{d\theta}{dt}\right]_{t=4} = [4t]_{t=4} = 16 \text{ rad/s}.$$

The angle is $\theta = [\text{mod}(2t^2, 2\pi)]_{t=4} = \text{mod}(32, 2\pi) = 0.5841$ rad, where $\text{mod}(x, y)$ ("modulus") is a standard function that returns the remainder of division of the first argument by the second. From which,

$$\boxed{\begin{aligned}\left[\frac{d\mathbf{e}}{dt}\right]_{t=4} &= 16\left(\mathbf{i}\cos\left(0.5841 + \frac{\pi}{2}\right) + \mathbf{j}\sin\left(0.5841 + \frac{\pi}{2}\right)\right)\\ &= -8.823\mathbf{i} + 13.35\mathbf{j}\end{aligned}}$$