PROBLEM: 002

GIVEN:

The acceleration of a point is $a = 20t$ m/s^2. When $t = 0$, $s = 40$ m and $v = -10$ m/s. What are the position and velocity at $t = 3$ s?

REQUIRED:

SOLUTION:

Solution: The velocity is

$$v = \int a\,dt + C_1,$$

where C_1 is the constant of integration. Thus

$$v = \int 20t\,dt + C_1 = 10t^2 + C_1.$$

At $t = 0$, $v = -10$ m/s, hence $C_1 = -10$ and the velocity is $v = 10t^2 - 10$ m/s. The position is

$$s = \int v\,dt + C_2,$$

where C_2 is the constant of integration.

$$s = \int (10t^2 - 10)\,dt + C_2 = \left(\frac{10}{3}\right)t^3 - 10t + C_2.$$

At $t = 0$, $s = 40$ m, thus $C_2 = 40$. The position is

$$s = \left(\frac{10}{3}\right)t^3 - 10t + 40 \text{ m}.$$

At $t = 3$ seconds,

$$s = \left[\frac{10}{3}t^3 - 10t + 40\right]_{t=3} = 100 \text{ m}.$$

The velocity at $t = 3$ seconds is

$$v = \left[10t^2 - 10\right]_{t=3} = 80 \text{ m/s}.$$