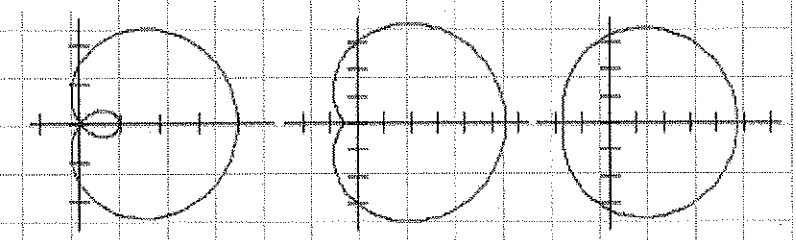
NAME DATE

SOLUTION

PROBLEM: AP-10

GIVEN:

A particle travels around a limaçon, defined by the equation $r = b - a \cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

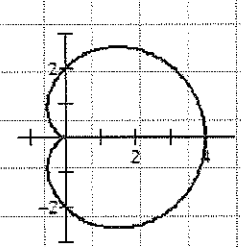


$b < a$ looped | $a < b < 2a$ dimpled | $2a \le b$ convex

$r = b + a\cos\theta$

REQUIRED:

SOLUTION:



$r = a \pm a\cos\theta$

$\theta = 3t^2$
$\dot\theta = 6t$
$\ddot\theta = 6$

$$r = b - a\cos\theta$$

$$\dot r = a\sin\theta\,\dot\theta$$

$$\ddot r = a\cos\theta\,\dot\theta^2 + a\sin\theta\,\ddot\theta$$

$$v_r = \dot r = a(\sin\theta)(\dot\theta)$$

$$v_\theta = r\dot\theta = (b - a\cos\theta)(\dot\theta)$$

$$a_r = \ddot r - r\dot\theta^2 = \left[a(\cos\theta)(\dot\theta)^2 + a(\sin\theta)(\ddot\theta)\right] - (b - a(\cos\theta))\dot\theta^2$$

$$= (2a(\cos\theta) - b)\dot\theta^2 + a(\sin\theta)\ddot\theta$$

$$a_\theta = r\ddot\theta + 2\dot r\dot\theta = (b - a(\cos\theta))\ddot\theta + 2(a\sin\theta\dot\theta)\dot\theta$$

$$= 2a(\sin\theta)\dot\theta^2 + (b - a(\cos\theta))\ddot\theta$$