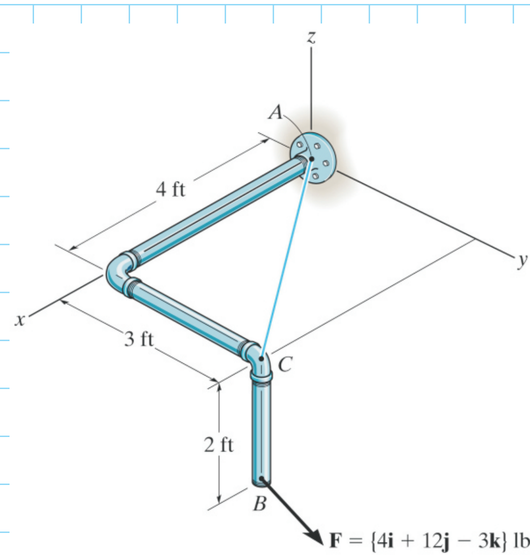
Problem AP-20

Given:

Determine the magnitude of the moments of the force **F** about the *x*, *y*, and *z* axes.

Determine the moment of the force **F** about an axis extending between *A* and *C*. Express the result as a Cartesian vector.



Required:

Solution:

Position Vector:

$$\mathbf{r}_{AB} = \{(4-0)\,\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Moment of Force* F *About x, y, and z Axes: The unit vectors along *x*, *y*, and *z* axes are **i**, **j**, and **k** respectively. Applying Eq. 4–11, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - (4)(3)] = 36.0\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

Position Vector:

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Unit Vector Along AC Axis:

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

Moment of Force* F *About AC Axis: With $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$ lb, applying Eq. 4–7, we have

$$M_{AC} = \mathbf{u}_{AC}\cdot(\mathbf{r}_{CB}\times\mathbf{F}) = \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4\text{ lb}\cdot\text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) = \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4\text{ lb}\cdot\text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

$$\mathbf{M}_{AC} = M_{AC}\,\mathbf{u}_{AC} = 14.4(0.8\mathbf{i} + 0.6\mathbf{j}) = \{11.5\mathbf{i} + 8.64\mathbf{j}\}\text{ lb}\cdot\text{ft}$$