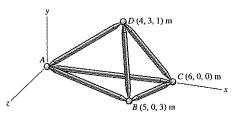
Problem 2.72 Determine the components of the position vector \mathbf{r}_{BD} from point B to point D. Use your result to determine the distance from B to D.



Solution: We have the following coordinates: A(0,0,0), B(5,0,3) m, C(6,0,0) m, D(4,3,1) m

$$\mathbf{r}_{BD} = (4 \text{ m} - 5 \text{ m})\mathbf{i} + (3 \text{ m} - 0)\mathbf{j} + (1 \text{ m} - 3 \text{ m})\mathbf{k}$$

$$= (-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \text{ m}$$

$$\mathbf{r}_{BD} = \sqrt{(-1 \text{ m})^2 + (3 \text{ m})^2 + (-2 \text{ m})^2} = 3.74 \text{ m}$$

Problem 2.73 What are the direction cosines of the position vector \mathbf{r}_{BD} from point B to point D?

Solution:

$$\cos \theta_x = \frac{-1 \text{ m}}{3.74 \text{ m}} = -0.267, \quad \cos \theta_y = \frac{3 \text{ m}}{3.74 \text{ m}} = 0.802,$$
$$\cos \theta_z = \frac{-2 \text{ m}}{3.74 \text{ m}} = 0.535$$

Problem 2.74 Determine the components of the unit vector \mathbf{e}_{CD} that points from point C toward point D.

Solution: We have the following coordinates: A(0,0,0), B(5,0,3) m, C(6,0,0) m, D(4,3,1) in

$$r_{CD} = (4 \text{ m} - 6 \text{ m})\mathbf{i} + (3 \text{ m} - 0)\mathbf{j} + (1 \text{ m} - 0)\mathbf{k} = (-2\mathbf{i} + 3\mathbf{j} + 1\mathbf{k})$$

$$r_{CD} = \sqrt{(-2 \text{ m})^2 + (3 \text{ m})^2 + (1 \text{ m})^2} = 3.74 \text{ m}$$

Thus

$$e_{CD} = \frac{1}{3.74 \text{ m}} (-2i + 3j + k) \text{ m} = (-0.535i + 0.802j + 0.267k)$$

Problem 2.75 What are the direction cosines of the unit vector \mathbf{e}_{CD} that points from point C toward point D?

Solution: Using Problem 2.74

$$\cos \theta_x = -0.535$$
, $\cos \theta_y = 0.802$, $\cos \theta_z = 0.267$

Problem 2.94 Consider the tower described in Problem 2.93. The magnitude of the force \mathbf{F}_{AB} is 2 kN. The x and z components of the vector sum of the forces exerted on the tower by the three cables are zero. What are the magnitudes of \mathbf{F}_{AC} and \mathbf{F}_{AD} ?

Solution: From the solution of Problem 2.93, the unit vectors are:

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = -\frac{40}{90}\mathbf{i} - \frac{70}{90}\mathbf{j} + \frac{40}{90}\mathbf{k}$$

= -0.4444i - 0.7778j + 0.4444k

$$u_{AD} = \frac{r_{AD}}{|r_{AD}|} = \frac{-60}{110}i - \frac{70}{110}j - \frac{60}{110}$$

$$= -0.5455 \mathbf{i} - 0.6364 \mathbf{j} - 0.5455 \mathbf{k}$$

From the solution of Problem 2.93 the force \mathbf{F}_{AB} is

$$\mathbf{F}_{AB} = |\mathbf{F}_{AB}|\mathbf{u}_{AB} = 0.9926\mathbf{i} - 1.737\mathbf{j} + 0\mathbf{k}$$

The forces \mathbf{F}_{AC} and \mathbf{F}_{AD} are:

$$\mathbf{F}_{AC} = |\mathbf{F}_{AC}|\mathbf{u}_{AC} = |\mathbf{F}_{AC}|(-0.4444\mathbf{i} - 0.7778\mathbf{j} + 0.4444\mathbf{k})$$

$$\mathbf{F}_{AD} = |\mathbf{F}_{AD}|\mathbf{u}_{AD} = |\mathbf{F}_{AD}|(-0.5455\mathbf{i} - 0.6364\mathbf{j} - 0.5455\mathbf{k})$$

Taking the sum of the forces:

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} = (0.9926 - 0.4444 |\mathbf{F}_{AC}| - 0.5455 |\mathbf{F}_{AD}|)i$$

+
$$(-1.737 - 0.7778|\mathbf{F}_{AC}| - 0.6364|\mathbf{F}_{AD}|)\mathbf{j}$$

$$+ (0.4444|\mathbf{F}_{AC}| - 0.5455|\mathbf{F}_{AD}|)\mathbf{k}$$

The sum of the x- and z-components vanishes, hence the set of simultaneous equations:

$$0.4444|\mathbf{F}_{AC}| + 0.5455|\mathbf{F}_{AD}| = 0.9926$$
 and

$$0.4444|\mathbf{F}_{AC}|-0.5455|\mathbf{F}_{AD}|=0$$

These can be solved by means of standard algorithms, or by the use of commercial packages such as TK Solver Plus ® or Mathead®. Here a hand held calculator was used to obtain the solution:

$$|\mathbf{F}_{AC}| = 1.1163 \text{ kN}$$

$$|F_{AD}| = 0.9096 \text{ kN}$$

Problem 2.95 In Example 2.10, suppose that the distance from point C to the collar A is increased from 0.2 m to 0.3 m, and the magnitude of the force T increases to 60 N. Express T in terms of its components.

Solution: The position vector from C to A is now $\mathbf{r}_{CA} = (0.3 \text{ m})\mathbf{e}_{CD} = (-0.137\mathbf{i} - 0.205\mathbf{j} + 0.171\mathbf{k})m$ The position vector form the origin to A is

$$\mathbf{r}_{OA} = \mathbf{r}_{OC} + \mathbf{r}_{CA} = (0.4\mathbf{i} + 0.3\mathbf{j}) \text{ m} + (-0.137\mathbf{i} - 0.205\mathbf{j} + 0.171\mathbf{k}) \text{ m}$$

$$\mathbf{r}_{OA} = (0.263\mathbf{i} + 0.0949\mathbf{j} + 0.171\mathbf{k}) \; \mathrm{m}$$

The coordinates of A are (0.263, 0.0949, 0.171) m. The position vector from A to B is

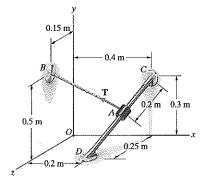
$$\mathbf{r}_{AB} = ([0 - 0.263]\mathbf{i} + [0.5 - 0.0949]\mathbf{j} + [0.15 - 0.171]\mathbf{k})$$
 m

$$\mathbf{r}_{AB} = (-0.263\mathbf{i} + 0.405\mathbf{j} - 0.209\mathbf{k})$$
 m

The force T is

$$T = (60 \text{ N}) \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (-32.7\mathbf{i} + 50.3\mathbf{j} - 2.60\mathbf{k}) \text{ N}$$

$$T = (-32.7i + 50.3j - 2.60k) N$$



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Solution: We know $\mathbf{r}_{BA} = 3.76\mathbf{i} - 2.63\mathbf{j} - 3\mathbf{k}$ m from Problem 2.97. The unit vector $\mathbf{u}_{AB} = -r_{BA}/|r_{BA}|$. The unit vector is $\mathbf{u}_{AB} = -0.686\mathbf{i} + 0.480\mathbf{j} + 0.547\mathbf{k}$. Hence, the force vector T is given by

T = |T|(-0.686i + 0.480j + 0.547k) N = -41.1i + 28.8j + 32.8k N

Problem 2.99 In Active Example 2.11, suppose that the vector **V** is changed to V = 4i - 6j - 10k.

Solution: From Active Example 2.4 we have the expression for U. Thus

- (a) What is the value of U-V?
- (b) What is the angle between **U** and **V** when they are placed tail to tail?

$$U = 6i - 5j - 3k, V = 4i - 6k - 10k$$

$$\mathbf{U} \cdot \mathbf{V} = (6)(4) + (-5)(-6) + (-3)(-10) = 84$$

$$\cos\theta = \frac{\mathbf{U} \cdot \mathbf{V}}{|\mathbf{V}||\mathbf{V}|} = \frac{84}{\sqrt{6^2 + (-5)^2 + (-3)^2} \sqrt{4^2 + (-6)^2 + (-10)^2}} = 0.814$$

$$\theta = \cos^{-1}(0.814) = 35.5^{\circ}$$

(a)
$$U \cdot V = 84$$
, (b) $\theta = 35.5^{\circ}$

Problem 2.100 In Example 2.12, suppose that the coordinates of point B are changed to (6, 4, 4) m. What is the angle θ between the lines AB and AC?

Solution: Using the new coordinates we have

$$\mathbf{r}_{AB}=(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$$
m, $|\mathbf{r}_{AB}|=3$ m

$$\mathbf{r}_{AC} = (4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) \text{ m}, |\mathbf{r}_{AC}| = 6.71 \text{ m}$$

$$\cos\theta = \frac{r_{AB} \cdot r_{AC}}{|r_{AB}||r_{AC}|} = \frac{((2)(4) + (1)(5) + (2)(2)) \text{ tr}^2}{(3 \text{ tn})(6.71 \text{ m})} = 0.845$$

$$\theta = \cos^{-1}(0.845) = 32.4^{\circ}$$

$$\theta = 32.4^{\circ}$$

 $\begin{array}{c}
C \\
(8, 8, 4) \text{ m} \\
A & \theta \\
(4, 3, 2) \text{ m} & (6, 1, -2) \text{ m} \\
\lambda
\end{array}$

Problem 2.101 What is the dot product of the position vector $\mathbf{r} = -10\mathbf{i} + 25\mathbf{j}$ (m) and the force vector

$$F = 300i + 250j + 300k$$
 (N)?

Solution: Use Eq. (2.23).

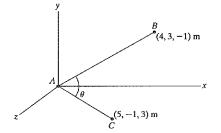
$$\mathbf{F} \cdot \mathbf{r} = (300)(-10) + (250)(25) + (300)(0) = 3250 \text{ N-m}$$

Problem 2.102 Suppose that the dot product of two vectors **U** and **V** is $\mathbf{U}\cdot\mathbf{V}=0$. If $|\mathbf{U}|\neq 0$, what do you know about the vector \mathbf{V} ?

Solution:

Either |V| = 0 or $V \perp U$

Problem 2.108 Determine the angle θ between the lines AB and AC (a) by using the law of cosines (see Appendix A); (b) by using the dot product.



Solution:

(a) We have the distances:

$$AB = \sqrt{4^2 + 3^2 + 1^2}$$
 m = $\sqrt{26}$ m

$$AC = \sqrt{5^2 + 1^2 + 3^2}$$
 m = $\sqrt{35}$ m

$$BC = \sqrt{(5-4)^2 + (-1-3)^2 + (3+1)^2}$$
 m = $\sqrt{33}$ m

The law of cosines gives

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\theta$$

$$\cos\theta = \frac{AB^2 + AC^2 - BC^2}{2(AB)(AC)} = 0.464 \quad \Rightarrow \boxed{\theta = 62.3^{\circ}}$$

(b) Using the dot product

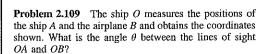
$$r_{AB} = (4i + 3j - k) \text{ m}, \quad r_{AC} = (5i - j + 3k) \text{ m}$$

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (4 \text{ m})(5 \text{ m}) + (3 \text{ m})(-1 \text{ m}) + (-1 \text{ m})(3 \text{ m}) = 14 \text{ m}^2$$

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (AB)(AC)\cos\theta$$

Therefore

$$\cos \theta = \frac{14 \text{ m}^2}{\sqrt{26} \text{ m}\sqrt{35} \text{ m}} = 0.464 \Rightarrow \theta = 62.3^{\circ}$$



Solution: From the coordinates, the position vectors are:

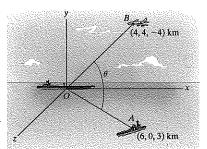
$$\mathbf{r}_{OA}=6\mathbf{i}+0\mathbf{j}+3\mathbf{k}$$
 and $\mathbf{r}_{OB}=4\mathbf{i}+4\mathbf{j}-4\mathbf{k}$

The dot product:
$$\mathbf{r}_{OA} \cdot \mathbf{r}_{OB} = (6)(4) + (0)(4) + (3)(-4) = 12$$

The magnitudes: $|r_{OA}| = \sqrt{6^2 + 0^2 + 3^2} = 6.71$ km and

$$|\mathbf{r}_{OA}| = \sqrt{4^2 + 4^2 + 4^2} = 6.93 \text{ km}.$$

From Eq. (2.24) $\cos\theta = \frac{\mathbf{r}_{OA} \cdot \mathbf{r}_{OB}}{|\mathbf{r}_{OA}||\mathbf{r}_{OB}|} = 0.2581$, from which $\theta = \pm 75^\circ$. From the problem and the construction, only the positive angle makes sense, hence $\theta = 75^\circ$



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Problem 2.115 Consider the cables AB and AC shown in Problem 2.114. Let \mathbf{r}_{AB} be the position vector from point A to point B. Determine the vector component of \mathbf{r}_{AB} parallel to the cable AC.

Solution: From Problem 2.114, $\mathbf{r}_{AB}=0$ i -7j + 10k, and $\mathbf{e}_{AC}=0.6667$ i - 0.3333j + 0.6667k. Thus $\mathbf{r}_{AB}\cdot\mathbf{e}_{AC}=9$, and $(\mathbf{r}_{AB}\cdot\mathbf{e}_{AC})\mathbf{e}_{AC}=(6\mathbf{i}-3\mathbf{j}+6\mathbf{k})$ ft.

Problem 2.116 The force $\mathbf{F} = 10\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ (N). Determine the vector components of \mathbf{F} parallel and normal to line OA.

(0, 6, 4) m F

Solution: Find
$$e_{OA} = \frac{r_{OA}}{|r_{OA}|}$$

Then

$$\mathbf{F}_P = (\mathbf{F} \cdot \mathbf{e}_{OA})\mathbf{e}_{OA}$$

and
$$\mathbf{F}_N = \mathbf{F} - \mathbf{F}_P$$

$$e_{OA} = \frac{0i + 6j + 4k}{\sqrt{6^2 + 4^2}} = \frac{6j + 4k}{\sqrt{52}}$$

$$e_{\mathit{OA}} = \frac{6}{7.21} \mathbf{j} + \frac{4}{7.21} \mathbf{k} = 0.832 \mathbf{j} + 0.555 \mathbf{k}$$

$$\mathbf{F}_P = [(10\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) \cdot (0.832\mathbf{j} + 0.555\mathbf{k})]\mathbf{e}_{OA}$$

$$F_P = [6.656]e_{OA} = 0i + 5.54j + 3.69k (N)$$

$$\mathbf{F}_{N}=\mathbf{F}-\mathbf{F}_{P}$$

$$\mathbf{F}_N = 10\mathbf{i} + (12 - 5.54)\mathbf{j} + (-6 - 3.69\mathbf{k})$$

$$F_N = 10i + 6.46j - 9.69k N$$

Problem 2.125 Two vectors U = 3i + 2j and V = 2i + 4j.

Solution: Use Eq. (2.34) and expand into 2 by 2 determinants.

(a) What is the cross product $U \times V$?

(b) What is the cross product $\mathbf{V} \times \mathbf{U}$?

$$\mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 2 & 4 & 0 \end{vmatrix} = \mathbf{i}((2)(0) - (4)(0)) - \mathbf{j}((3)(0) - (2)(0))$$

$$+ k((3)(4) - (2)(2)) = 8k$$

$$\mathbf{V} \times \mathbf{U} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \mathbf{i}((4)(0) - (2)(0)) - \mathbf{j}((2)(0) - (3)(0))$$

$$+ k((2)(2) - (3)(4)) = -8k$$

Problem 2.126 The two segments of the L-shaped bar are parallel to the x and z axes. The rope AB exerts a force of magnitude. $|\mathbf{F}| = 500$ lb on the bar at A. Determine the cross product $\mathbf{r}_{CA} \times \mathbf{F}$, where \mathbf{r}_{CA} is the position vector form point C to point A.

Solution: We need to determine the force F in terms of its components. The vector from A to B is used to define F.

$$\mathbf{r}_{AB} = (2\mathbf{i} - 4\mathbf{j} - \mathbf{k}) \mathbf{f}$$

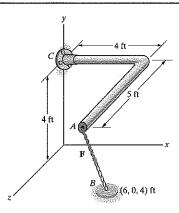
$$\mathbf{F} = (500 \text{ lb}) \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (500 \text{ lb}) \frac{(2\mathbf{i} - 4\mathbf{j} - \mathbf{k})}{\sqrt{(2)^2 + (-4)^2 + (-\mathbf{i})^2}}$$

$$\mathbf{F} = (218\mathbf{i} - 436\mathbf{j} - 109\mathbf{k})$$
 lb

Also we have $r_{CA} = (4i + 5k)$ ft

$$\mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 5 \\ 218 & -436 & -109 \end{vmatrix} = (2180\mathbf{i} + 1530\mathbf{j} - 1750\mathbf{k}) \text{ ft-lb}$$

$$\mathbf{r}_{CA} \times \mathbf{F} = (2180\mathbf{i} + 1530\mathbf{j} - 1750\mathbf{k}) \text{ ft-lb}$$



Problem 2.136 The cable BC exerts a 1000-lb force F on the hook at B. Determine $\mathbf{r}_{AB} \times F$.

Solution: The coordinates of points A, B, and C are A (16, 0, 12), B (4, 6, 0), C (4, 0, 8). The position vectors are

 $\mathbf{r}_{\mathit{OA}} = 16\mathbf{i} + 0\mathbf{j} + 12\mathbf{k}, \\ \mathbf{r}_{\mathit{OB}} = 4\mathbf{i} + 6\mathbf{j} + 0\mathbf{k}, \\ \mathbf{r}_{\mathit{OC}} = 4\mathbf{i} + 0\mathbf{j} + 8\mathbf{k}.$

The force F acts along the unit vector

$$e_{BC} = \frac{r_{BC}}{|r_{BC}|} = \frac{r_{OC} - r_{OB}}{|r_{OC} - r_{OB}|} = \frac{r_{AB}}{|r_{AB}|}$$

Noting
$$\mathbf{r}_{OC} - \mathbf{r}_{OB} = (4 - 4)\mathbf{i} + (0 - 6)\mathbf{j} + (8 - 0)\mathbf{k} = 0\mathbf{i} - 6\mathbf{j} + 8\mathbf{k}$$

$$|\mathbf{r}_{OC} - \mathbf{r}_{OB}| = \sqrt{6^2 + 8^2} = 10$$
. Thus

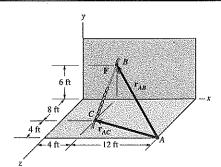
$$e_{BC} = 0i - 0.6j + 0.8k, \ \ {\rm and} \ \ F = |F|e_{BC} = 0i - 600j + 800k \ \ (lb).$$

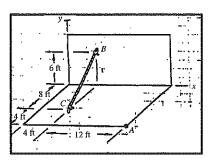
The vector

$$\mathbf{r}_{AB} = (4 - 16)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 12)\mathbf{k} = -12\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$$

Thus the cross product is

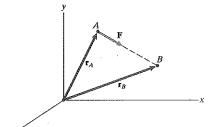
$$\mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -12 & 6 & -12 \\ 0 & -600 & 800 \end{vmatrix} = -2400\mathbf{i} + 9600\mathbf{j} + 7200\mathbf{k} \text{ (ft-lb)}$$





Problem 2.137 The force vector **F** points along the straight line from point A to point B. Its magnitude is $|\mathbf{F}| = 20 \text{ N}$. The coordinates of points A and B are $x_A = 6 \text{ m}$, $y_A = 8 \text{ m}$, $z_A = 4 \text{ m}$ and $x_B = 8 \text{ m}$, $y_B = 1 \text{ m}$, $z_B = -2 \text{ m}$.

- (a) Express the vector \mathbf{F} in terms of its components.
- (b) Use Eq. (2.34) to determine the cross products $\mathbf{r}_A \times \mathbf{F}$ and $\mathbf{r}_B \times \mathbf{F}$.



Solution: We have $\mathbf{r}_A = (6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k})$ m, $\mathbf{r}_B = (8\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ m,

(b)
$$r_{A} \times \mathbf{F} = \frac{20 \text{ N}}{\sqrt{89}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 \text{ m} & 8 \text{ m} & 4 \text{ m} \\ 2 & -7 & -6 \end{vmatrix}$$

$$= (-42.4\mathbf{i} + 93.3\mathbf{j} - 123.0\mathbf{k}) \text{ Nm}$$

$$\mathbf{r}_{B} \times \mathbf{F} = \frac{20 \text{ N}}{\sqrt{89}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 \text{ m} & 1 \text{ m} & -2 \text{ m} \\ 2 & -7 & -6 \end{vmatrix}$$

$$= (-42.4\mathbf{i} + 93.3\mathbf{j} - 123.0\mathbf{k}) \text{ Nm}$$

Note that both cross products give the same result (as they must).