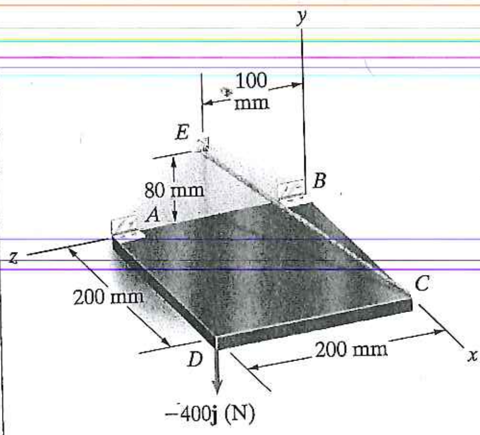
Example 5.9 Reactions at Properly Aligned Hinges (▶ *Related Problem 5.112*)



The plate is supported by hinges at A and B and the cable CE. The prop aligned hinges do not exert couples on the plate, and the hinge at A does not e a force on the plate in the direction of the hinge axis. Determine the react at the hinges and the tension in the cable.

Strategy

We will draw the free-body diagram of the plate, using the given informa about the reactions exerted by the hinges at A and B. Before the equilibr equations can be applied, we must express the force exerted on the plate by cable in terms of its components.

Solution

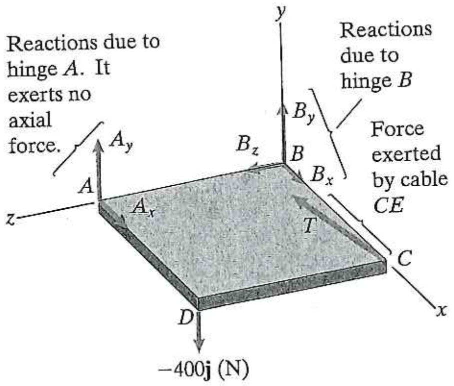
Draw the Free-Body Diagram We isolate the plate and show the react at the hinges and the force exerted by the cable (Fig. a). The term T is the f exerted on the plate by cable CE.

Apply the Equilibrium Equations Since we know the coordinates of pc C and E, we can express the cable force as the product of its magnitude T a unit vector directed from C toward E. The result is

$$T(-0.842\mathbf{i} + 0.337\mathbf{j} + 0.421\mathbf{k}).$$

The sums of the forces in each coordinate direction equal zero:

$$\Sigma F_x = A_x + B_x - 0.842T = 0,$$
$$\Sigma F_y = A_y + B_y + 0.337T - 400 = 0,$$
$$\Sigma F_z = B_z + 0.421T = 0.$$



(a) The free-body diagram of the plate.

If we sum the moments about B, the resulting equations will not contain three unknown reactions at B. The sum of the moments about B, with fo in N and distances in m, is

$$\Sigma \mathbf{M}_{\text{point } B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0 \\ -0.842T & 0.337T & 0.421T \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.2 \\ A_x & A_y & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0.2 \\ 0 & -400 & 0 \end{vmatrix}$$

$$= (-0.2A_y + 80)\mathbf{i} + (-0.0842T + 0.2A_x)\mathbf{j}$$
$$+ (0.0674T - 80)\mathbf{k} = 0.$$

The scalar equations are

$$\Sigma M_x = -(0.2 \text{ m})A_y + 80 \text{ N-m} = 0,$$
$$\Sigma M_y = -(0.0842 \text{ m})T + (0.2 \text{ m})A_x = 0,$$
$$\Sigma M_z = (0.0674 \text{ m})T - 80 \text{ N-m} = 0.$$

Solving these equations, we obtain the reactions

$$T = 1187 \text{ N}, \quad A_x = 500 \text{ N}, \quad A_y = 400 \text{ N}.$$

Then from Eqs. (1), the reactions at B are

$$B_x = 500 \text{ N}, \quad B_y = -400 \text{ N}, \quad B_z = -500 \text{ N}.$$